Outline of the presentation

- Classical operators do not allow a dynamic pruning with respect to both example coverage and language bias.
- Definition of new quasi-orders, natural relations, that allow so.
- Do ideal operators [van der Laag and Nienhuys-Cheng, 1994] exist for natural relations?
- Conclusion, Perspectives.
Inductive Logic Programming: Definite settings

Examples and hypotheses are definite clauses.

[Muggleton and Raedt, 1994]: Given $E^+$ (positive training examples) and $E^-$ (negative training examples) for a target concept, and a background knowledge $B$,
find a hypothesis $H$ such that

$$\forall e^+ \in E^+ : \ B \cup H \models e^+ \quad (H \text{ is complete})$$

$$\forall e^- \in E^- : \ B \cup H \not\models e^- \quad (H \text{ is consistent})$$

An illustrative example: the grand-father concept

\[
\begin{align*}
\text{pa}(A, B) & \leftarrow \text{f}(A, B) \\
\text{pa}(A, B) & \leftarrow \text{m}(A, B) \\
\text{gf}(\text{abraham}, \text{bart}) & \leftarrow \text{f}(\text{abraham}, \text{homer}), \text{f}(\text{homer}, \text{bart}) \\
\text{gf}(\text{grampa-bouvier}, \text{bart}) & \leftarrow \text{f}(\text{grampa-bouvier}, \text{marge}), \text{m}(\text{marge}, \text{bart}) \\
& \leftarrow \text{gf}(\text{mona}, \text{bart}), \text{m}(\text{mona}, \text{homer}), \text{f}(\text{homer}, \text{bart}) \\
& \leftarrow \text{gf}(\text{jackie}, \text{bart}), \text{m}(\text{jackie}, \text{marge}), \text{m}(\text{marge}, \text{bart}) \\
\text{gf}(A, B) & \leftarrow \text{f}(A, C), \text{pa}(C, B)
\end{align*}
\]
Refinement operator & Pruning

[Mitchell, 1982]: The search should respect a generality order to allow for pruning (with respect to example coverage).

Example: adding literal operator.

\[ g(f(A, B) \leftarrow m(A, C)) \]

\[ g(f(A, B) \leftarrow f(A, C)) \]

\[ g(f(A, B) \leftarrow \text{pa}(C, B)) \]

\[ g(f(A, B) \leftarrow m(A, C), \text{pa}(C, B)) \]

Language bias

- [Mitchell, 1991]: Bias is necessary for learning (quality of learning results and efficiency).
- Language bias: constraints on the hypotheses syntax.
  - range-restriction, connection,
  - bound on the size, on the number of variables, on the depth of terms, etc.
- Those biases do not make the search more efficient, as dynamic pruning with respect to those constraints is in general not possible.
**Example (fd)**

\[
gf(A, B) \leftarrow \\
gf(A, B) \leftarrow m(A, C) \\
gf(A, B) \leftarrow f(A, C) \\
gf(A, B) \leftarrow po(C, B) \\
\cdots \\
gf(A, B) \leftarrow f(A, C), po(C, B)
\]

Language bias: Range restriction.

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**Private properties**

**Definition 1** (*private property*) A property \(P\) is private with respect to a relation \(\mathcal{R}\) iff

\[
H \mathcal{R} H' \land \overline{P(H)} \Rightarrow P(H')
\]

\(\mathcal{R}\) = adding literal, \(P\) = coverage of \(e^+\) or \(P\) = size.
Natural relations

Natural relation of a property $P$: a largest relation for which $P$ is private (a relation that contains the most of different links).

- The natural relation of a property $P$ is unique and any relation that makes $P$ private is included in this natural relation.
- Two hypotheses $C$ and $D$ are in natural relation for a property $f(H) \subseteq f(D)$.

\[
H \models e^+ \quad \ldots \quad C \models D
\]

\[
|H| \leq |D|\]

Conjunctions of properties

$H$ $\theta$-subsumes $e^+$ and $|H| \leq k$.

$C$ $\theta$-subsumes $D$ and $|C| \leq |D|$.
Ideal operators

[van der Laag and Nienhuys-Cheng, 1994]

**Definition 2 (ideality)** An operator is ideal if it is locally finite, proper and complete.

- **Locally finite**: $O(H)$ is computable;
- **Proper**: $O(H)$ does not contain any clause equivalent to $H$;
- **Complete**: $O^*(H)$ contains all clauses comparable to $H$.

Ideal operators do not exist for $\theta$-subsumption or logical implication.

Covers

**Definition 3 (cover)** $C$ covers $D$ iff $C > D$ and there exists no $E$ such that $C > E > D$. $C$ is an upward cover of $D$. $D$ a downward cover of $C$.

**Definition 4 (cover set)** A downward (resp. upward) cover set of a clause $C$ is a maximal set of incomparable downward (resp. upward) covers of $C$.
Non-existence

An ideal operator computes at least a cover set.

Then, there are two possible problems.

1. The cover set is not defined, there is an uncovered infinite chain, no complete and computable operator exists.
2. The cover set is infinite, a complete operator cannot be computable.
\[
\begin{align*}
C: q(X_1) & \leftrightarrow p(X_1, X_1) , \\
D_n: q(X_1) & \leftrightarrow \{p(X_i, X_j) | 1 \leq i, j \leq n, i \neq j\} .
\end{align*}
\]

\[
D_2 : q(X_1) \leftrightarrow p(X_1, X_2), p(X_2, X_1)
\]

\[
D_3 : q(X_1) \leftrightarrow p(X_1, X_2), p(X_1, X_3), p(X_2, X_1), p(X_2, X_3), p(X_3, X_1), p(X_3, X_2)
\]

\[
D_n : q(X_1) \leftrightarrow \{p(X_i, X_j) | 1 \leq i, j \leq n, i \neq j\}
\]

\[
C: q(X_1) \leftrightarrow p(X_1, X_1)
\]
An ideal operator: $\rho^0(C)$

1. Add a literal with new variables to $C$.
2. Unify two variables $X_1$ and $X_2$ of $C$, such that $C >^\theta C\{X_1/X_2\}$.
   If the size of the result decreases, add literals with new variables.
3. Apply previous operations (1, 2) on clauses equivalent to $C$ ($\theta$-equivalent and same size).
4. Apply operation 1 on subsets of $C$ which are equivalent to $C$, one new literal at least must use a predicate symbol which does not appear in $C$.

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**Conclusion**

- Dynamic pruning with respect to language bias based on the use of new quasi orders: *natural relations*.
- Existence of ideal operators for unrestricted search spaces ordered by natural relations.

Related works:  
  - [Shapiro, 1981]  
  - [Champesme et al., 1995, Esposito et al., 1996]

Perspectives: consider other families of operators for spaces ordered by natural relations (*optimal* [De Raedt and Bruynooghe, 1993]).
References


